Way of Creation of Super Gain Formula

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Abstract. Several years ago HB9XM introduced a new (for me) equation to calculate the gain of a parabola dish antenna. The equation was frequently used during measurements on our radio telescope "Ricken Süd" but nobody was able to explain the theory behind that simple but nevertheless very useful equation. Here an appropriate theory is presented to understand how the equation was designed.

Key words. Beam solid angle, Wavelength, Effective area, Efficiency.

1. Theory

Starting from the scratch the antenna gain G (Kraus, 1965) can be calculated by simply dividing the sky solid angle 4π by the beam solid angle Ω_a of the antenna,

$$G = 4\pi/\Omega_a \tag{1}$$

where, again according to (Kraus, 1965)

$$\Omega_a = \lambda^2/A_e \tag{2}$$

A_e is a commonly used expression for the effective area of the antenna involved. Thus by introducing frequency f and speed of light in vacuum c the gain G can now be expressed by

$$G = 4\pi A_e \frac{f^2}{c^2} \tag{3}$$

If we think of a standard parabola antenna the effective aperture A_e can be calculated by knowing the efficiency factor η and the diameter D of the reflecting area of the mirror.

$$A_e = \eta \frac{\pi D^2}{4} \tag{4}$$

Combining Eqs. 1, 3 and 4 by straight forward analysis we immediately get

$$G = \eta \frac{\pi^2 D^2 f^2}{c^2} \tag{5}$$

For practical use of Eq. 5 we intend to put in the frequency f not in Hz but in GHz thus Eq. 5 has to be rewritten to

$$G = \eta \frac{\pi^2}{c^2}(10^9)^2 \ (D/m)^2 \ (f/GHz)^2 \tag{6}$$

Now the antenna gain G can also be expressed in dB by applying logarithm functions to Eq. 6

$$\begin{aligned} G_{dB} &= 10log(\eta) + 10log[(10^9)^2 \ \pi^2/c^2] + \\ &\quad + 10log[(D/m)^2] + 10log[(f/GHz)^2] \end{aligned} \tag{7}$$

If we put in all known constant values we then simply get

$$\begin{aligned} G_{dB} &= 10log(\eta) + 20.4dB + \\ &\quad + 20[log(D/m) + log(f/GHz)] \end{aligned} \tag{8}$$

2. Final result

Assuming that the efficiency factor $\eta = 0.54$ (54%) we can rewrite Eq. 8 to the well known super formula Eq. 9

$$G_{dB} = 17.7dB + 20[log(D/m) + log(f/GHz)] \tag{9}$$

That's all volks what I wanted to present. Personally I recommend to learn Eq. 8 by heart instead of Eq. 9 because it's probably more useful for daily practical work.

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References

John. D. Kraus, Radio Astronomy 2nd edition, Cygnus-Quasar Books 1986, p. 6-6, Eq. 6-17 until 6-22.